

## **ASED-AIM Analysis of EM Scattering by 3D Huge-Scale Finite Periodic Arrays**

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### ***Abstract***

*In this paper, the Adaptive Integral Method (AIM) has been successfully extended to characterize electromagnetic scattering by large scale finite periodic arrays with each cell comprising of either dielectric or metallic objects. During the course of investigation, accurate sub-entire- domain (ASED) basis function has been presented. The complexity analysis shows that the computational time of  $O(N_0 \log N_0) + O(M \log M)$  and memory requirement of  $O(N_0) + O(M)$  in the ASED-AIM.*

### **Introduction**

Large-scale finite periodic structures consisting of metamaterials [1] had been topics of great interest and it has instructional and clinical applications. Consequently, correct complete wave evaluation of these systems using numerical techniques including MoM are very vital. Whilst using MoM to remedy these troubles, the memory requirement and computational complexity are  $O(N^2)$  wherein  $n$  is range of unknowns. These days evolved speedy solvers consisting of intention [2]-[4] can alleviate the stringent necessities and reduce memory requirement to  $O(N)$  and computational complexity to  $O(N \log N)$  respectively. But, full wave simulations for those structures, which are very difficult because critical properties (which includes periodicity) aren't utilized in traditional solvers. These days, a few novel physics-primarily based basis functions have been proposed to remedy those difficult troubles together with accurate sub-entire-domain (ASED) foundation characteristic [5]. The use of ASED basis feature, the unit mobile may be represented with one foundation function; hence, the full quantity of unknowns can be added down from  $MN_0$  to  $N_0$  where  $N_0$  is wide variety of cells in the complete domain and  $m$  is variety of unknowns in one unit mobile. Consequently, ASED-intention, that's the aggregate of ASED with intention, can reduce the reminiscence requirement from  $O(MN_0)$  (the usage of conventional intention) to  $O(M) + O(N_0)$  and reduce the computational complexity from  $O(MN_0 \log MN_0)$  (the usage of traditional AIM) to  $O(M \log M) + O(N_0 \log N_0)$ .

For this purpose, numerical examples have been presented which validate the accuracy and performance of the proposed ASED-aim in solving large-scale finite periodic array issues.

## 2. ASED-AIM Formulation

Electromagnetic scattering by periodic composite conducting and dielectric objects can be characterized using volume-surface integral equation (VSIE):

$$\mathbf{E}^i(\mathbf{r}) = \mathbf{E}(\mathbf{r}) - \mathbf{E}^s(\mathbf{r}), \quad \mathbf{r} \in V; \quad \text{and} \quad \mathbf{E}^i(\mathbf{r})|_{\tan} = -\mathbf{E}^s(\mathbf{r})|_{\tan}, \quad \mathbf{r} \in S. \quad (1)$$

Equivalent electric volume and surface current  $\mathbf{J}_V(\mathbf{r})$  and  $\mathbf{J}_S(\mathbf{r})$  are related to total electric field  $\mathbf{E}(\mathbf{r})$  and scattered electric field  $\mathbf{E}^s(\mathbf{r})$  via

$$\mathbf{J}_V(\mathbf{r}) = j\omega\kappa\mathbf{D}(\mathbf{r}) = j\omega(\epsilon - \epsilon_0)\mathbf{E}(\mathbf{r}), \quad \mathbf{r} \in V \quad (2)$$

$$\begin{aligned} \mathbf{E}^s(\mathbf{r}) = & -j\omega\mu_0 \int_V G(\mathbf{r}, \mathbf{r}') \mathbf{J}_V(\mathbf{r}') dV' - j\omega\mu_0 \int_S G(\mathbf{r}, \mathbf{r}') \mathbf{J}_S(\mathbf{r}') dS' \\ & + \frac{\nabla}{j\omega\epsilon_0} \int_V G(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}_V(\mathbf{r}') dV' + \frac{\nabla}{j\omega\epsilon_0} \int_S G(\mathbf{r}, \mathbf{r}') \nabla' \cdot \mathbf{J}_S(\mathbf{r}') dS' \end{aligned} \quad (3)$$

where  $G(\mathbf{r}; \mathbf{r}')$  denotes free space Green's function,  $\mu_0$  and  $\epsilon_0$  represent free space permeability and permittivity respectively,  $\epsilon$  stands for permittivity in the dielectric object, and  $k = (\epsilon - \epsilon_0)/\epsilon$  identifies the contrast ratio of scatterer material and its background medium. For the  $p$ -th cell, surface currents and volume currents can be expanded as follows:

$$\mathbf{J}_p^S = \sum_{m=1}^{N_S} I_{pm}^S \mathbf{f}_{pm}^S, \quad \text{and} \quad \mathbf{J}_p^V = \sum_{m=1}^{N_V} I_{pm}^V \kappa \mathbf{f}_{pm}^V; \quad (4)$$

where  $\mathbf{f}_{pm}^S$  and  $\mathbf{f}_{pm}^V$  denote respectively the RWG and SWG basis functions associated with the  $m$ -th surface and volume basis functions of the  $p$ -th cell,  $N_S$  is the number of RWG basis functions while  $N_V$  is the number of SWG basis functions, and  $I_{pm}^S$  and  $I_{pm}^V$  stand for the respective unknown coefficients to be solved for. Thus, the current density can be written in terms of electric current for the  $p$ -th cell as follows:

$$\mathbf{J} = \sum_{p=1}^{N_0} j_p \mathbf{J}_p \quad \text{where} \quad \mathbf{J}_p = \mathbf{J}_p^S + \mathbf{J}_p^V; \quad (5)$$

where  $j_p$  denotes unknowns to be solved for. After solving a small array problem, we can construct ASED basis functions for each cell and then use them to solve the entire problem. The cell impedance matrix elements (denoted by the superscript  $C$  herein and subsequently) can be written as

$$Z_{pq}^C = \sum_{m=1}^M \sum_{n=1}^M I_{pm} Z_{pmqn} I_{qn}. \quad (6)$$

Using conventional AIM, the matrix vector multiplication can be written as

$$\overline{\mathbf{Z}}\mathbf{I} = \overline{\mathbf{V}}\overline{\mathbf{H}}\overline{\mathbf{P}}\mathbf{I} + \overline{\mathbf{Z}}^{\text{near}}\mathbf{I} \quad (7)$$

where  $\mathbf{V}$  is the interpolation matrix,  $\mathbf{H}$  is Green's function matrix, and  $\mathbf{P}$  is the projection matrix. The four steps of conventional AIM can be shown in Fig. 1(a). For the far zone where  $\mathbf{V}$  is the interpolation matrix,  $\mathbf{H}$  is Green's function matrix, and  $\mathbf{P}$  is the projection matrix. The four steps of conventional AIM can be shown in Fig. 1(a). For the far zone interaction, the impedance matrix elements can be approximated as:

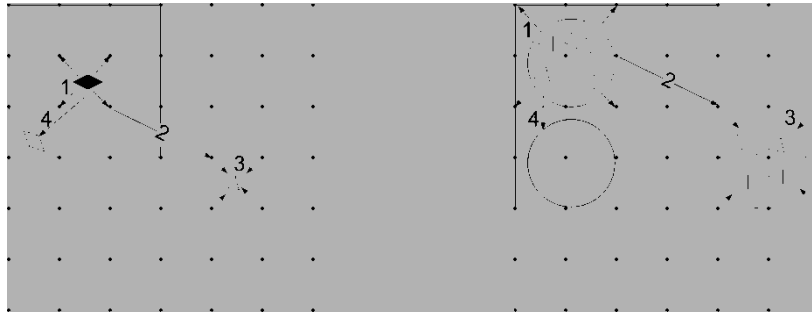


Figure 1: The pictorial representation of (a) conventional AIM and (b) ASED-AIM.

$$Z_{p_m q_n} \approx \tilde{Z}_{p_m q_n} = \sum_s \sum_t V_{m_s} H_{m_s n_t} P_{n_t} \quad (8)$$

where  $\Sigma$  denotes summation of all the grids associated with the basis functions. Thus, for cell interaction in the far zone, we have

$$Z_{pq}^C = \sum_m \sum_n I_{p_m} Z_{p_m q_n} I_{q_n} \approx \sum_m \sum_s \sum_n \sum_t I_{p_m} V_{m_s} H_{m_s n_t} I_{q_n} P_{n_t} = V_p^C H_{pq} P_q^C \quad (9)$$

where  $V^C$  and  $P^C$  are the interpolation and projection matrices for cell basis functions. They can be written explicitly as:

$$V_p^C = \sum_m \sum_s I_{p_m} V_{m_s}, \quad \text{and} \quad P_q^C = \sum_n \sum_t I_{q_n} P_{n_t}. \quad (10)$$

Now, Using ASED-AIM, the matrix vector multiplication can be written as

$$\overline{\mathbf{Z}}^C \cdot \overline{\mathbf{I}}^C = \overline{\mathbf{V}}^C \cdot \overline{\mathbf{H}} \cdot \overline{\mathbf{P}}^C \cdot \overline{\mathbf{I}}^C + \overline{\mathbf{Z}}^{C,\text{near}} \cdot \overline{\mathbf{I}}^C \quad (11)$$

The four steps for implementing ASED-AIM is shown graphically in Fig. 1(b).

### 3. Numerical Results

In this section, several examples will be given to demonstrate the validity and efficiency of our code to solve electromagnetic scattering by large scale periodic structures consisting of composite metallic and dielectric objects. In all the examples, the periodicity of the arrays in  $x$ -,  $y$ -, and  $z$ -directions are all  $0.2\lambda$ . The arrays are under normal incidence at  $\theta_i = 0^\circ$  and  $\phi^i = 0^\circ$  with electric field theta-polarized. First, we consider a 2D periodic structure shown in

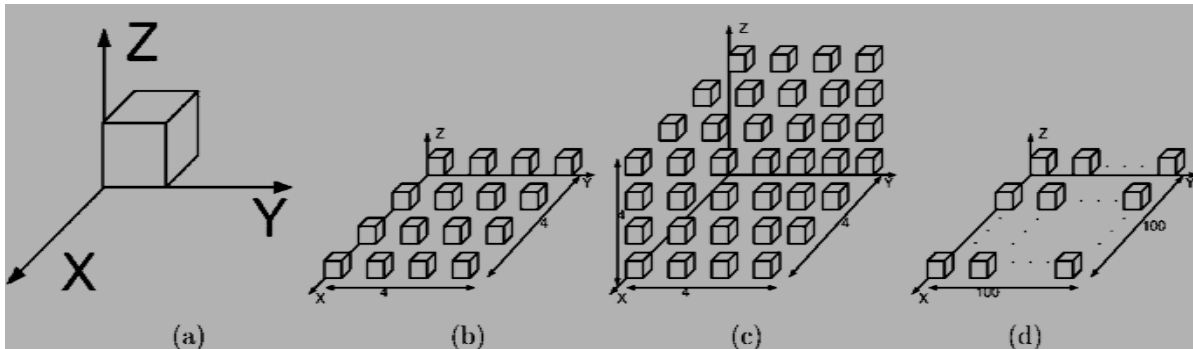


Figure 2: Examples of arrays used in the calculations of numerical results. (a) The structure of a unit cell,  $d = 0.2\lambda_0$ . The shaded area denotes a metallic patch while cube is a dielectric object with  $\epsilon = 2.2$ . (b)  $4 \times 4$  array. (c)  $4 \times 4 \times 4$  array. (d)  $100 \times 100$  array.

Fig. 2(b), which is a  $4 \times 4$  array in  $xy$  plane. The results shown in Fig. 3(a) are generated by ASSED-AIM and conventional AIM. Excellent agreement has been observed. Subsequently, we investigate the computational complexity and memory requirement of ASSED-AIM and compare them with those of the conventional AIM. Figs. 4(a) and (b) respectively show the relationship between the computational time and memory requirement with the number of unknowns using ASSED-AIM and AIM. From these figures, it is clear that ASSED-AIM is much more efficient in solving periodic array problems than AIM. The above method can be also extended to analyze 3D finite periodic structures. In Fig. 3(b), the RCS values calculated using ASSED-AIM and AIM are compared for the  $4 \times 4 \times 4$  array shown in Fig. 2(c) and good agreement have been observed. Finally, we consider an electrically very large finite periodic structure with  $100 \times 100$  array shown in Fig. 2(d). The total number of unknowns in this example is 10.87 million. The calculated radar cross section is shown in Fig. 3(c). For such an electrically large structure with over 10 million unknowns, ASSED-AIM only requires 273 MB memory and 1200 seconds, which demonstrates the efficiency of the new method in solving problems of electromagnetic scattering by large-scale periodic structures.

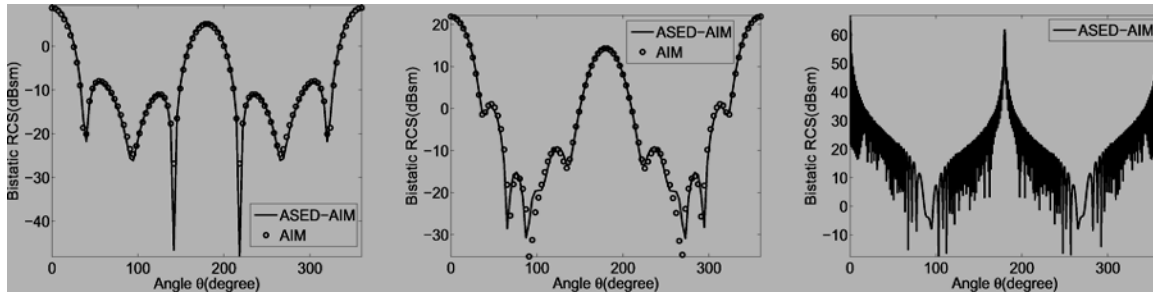


Figure 3: Bistatic RCS of the (a)  $4 \times 4$  array, (b)  $4 \times 4 \times 4$  array, and (c)  $100 \times 100$  array (leading to over 10 million unknowns); with each cell shown in Fig. 2(a).

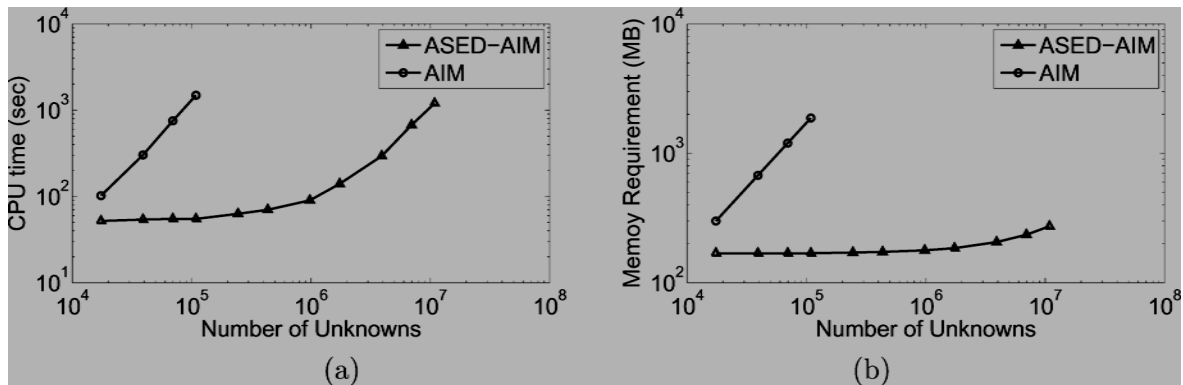


Figure 4: The relationship between (a) computational time (b) memory requirement and the number of unknowns using ASEDAIM (triangle line) and AIM (circle line).

#### 4. Conclusions

In this paper, a new algorithm integrating the ASED basis functions into the AIM has been developed to solve problems of scattering by huge-scale finite periodic arrays comprising of metallic and dielectric objects. It has thus reduced the memory requirement and computational time significantly in solving the array problems. High accuracy and efficiency of the ASEDAIM has been demonstrated. An example with over 10 million unknowns is successfully considered in a personal computer and its numerical results are illustrated.

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